"Quantum networks": a new approach for representing a network and evaluating hydraulic and thermal losses in district heating/cooling systems

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Abstract:

District heating networks (DHN) are becoming an energy-saving and cost-effective alternative to conventional conversion systems (boilers, chillers, combined heating and power systems) and appear as a vector of the energy transition because of the diversity of the energy sources that can be used for their operation. They are increasingly used in cogeneration and more efficiently when operating at lower temperature levels. Indeed, many previous works were able to simulate thermal and hydraulic losses of different structures of networks. These works have been carried out in several European countries, notably in Switzerland, Great Britain but also in countries such as Denmark and Sweden. The approach for representing the networks and the simulation models could be complex to use for a general optimization purpose.

This paper presents a new approach and methodology to represent the structure of the network in order to simplify models while performing accurate calculations for the thermal and hydraulic losses in the systems. The key parameters for the design and operation of the advanced heating/cooling network such as the nominal capacity of the network and the range of the working conditions (flow rates, minimum heating/cooling capacity, differential temperature on the substations) can be easily defined in hourly basis.

A typical application for a set of high-performance buildings in the BlueFACTORY district (Fribourg, Switzerland) has been considered to simulate different structures (branched, mesh or mixed network systems) of advanced thermal networks. The performances, in term of pressure drop and thermal losses on the pipes, are determined in function of the geometric and operational parameters of the network. It includes the configurations of decentralized substation units (heat exchangers, heat pumps, or mixed substations) for heating and hot water production.

Keywords:

ECOS Conference, Energy systems, District heating/cooling networks, Methodolog modelling simulation and analysis.

1. Introduction

Only 5% of houses in Switzerland are currently heated by a district heating (DH) system. The energy strategy 2050 [1] set up by the swiss confederation aims to promote the use of renewable energy sources, and the DH networks are clearly in line with this, as discussed by Lund et al. [2], Averfalk and Werner [3] et Buffa et al. [4]. As this technology is increasingly used, as Werner [5] explains, it is essential to use an efficient method to optimize these networks and thus avoid generating too much energy loss due to incorrect sizing. In order to make this optimization effective, a method of discretization of the network to quickly obtain a balance of mass flow, pressure drops, and heat losses must be used.

The first network modelling methods were developed in Germany by Loewen [6] and in Denmark by Pálsson et al. [7]. These two methods have been repeatedly used and discussed in different works [8-10]. These two approaches suggest transforming a tree structure into a linear structure and simplifying the short branches. However, these methods do not allow to deal with possible loops (the German method replaces them with serial lines). In addition, it is not possible to conserve certain parameters

such as the inside diameter of the pipe, heat losses or pressure drops depending on the methods. These two approaches are used mainly to determine the load profile of the plant.

The approach presented by Vesterlund and Dahl [9] does not simplify the network into a vector and therefore allows the interpretation of loops. Their work is focused on ensuring a high flexibility of the model by facilitating the addition of substations or sources on the network. The model calculates the distribution of mass flows in the network pipes by considering the local pressure at the intersections of the pipes, which is determined as a function of the pressure drop coefficients of the pipes and the mass flows of the pipes themselves. This "nodal" method has the advantage of retaining all the network parameters and is very modular. It also includes the fact of grouping several substations into a group that is now just one substation (a district of a city) in order to reduce the computing time. However, the level of discretization of the network remains quite general and this requires more resources for the optimization phase as discussed by Åberg and Widén [11] or Bøhm et al. [12].

Wang et al. [10] adopt the principle of nodal discretization of the network by adding a third dimension to the model in order to separate the supply and return pipes. The results are then processed in the matrix form. A distinction is made between a "supply" matrix and a "return" matrix. These are interconnected by the consumers or producers of heat. However, it is difficult to calibrate these matrices without using a genetic algorithm and the optimization process would require large temporal resources if the size of the network was significant.

Currently, four methods have been identified by Sarbu et al. [13] : loop, node, pipe and mixed nodeloop methods. The loop method is used for hydraulic analysis of network that use corrective flow rates as unknown. In nodal analysis, the unknowns are usually the pressure heads at the network nodes, and in the pipe method, the unknowns are the pipe flow rates. None of these methods focuses on an innovative way of representing and interpreting a network. The components considered are essentially branches and nodes.

2. Methodology

2.1. Network structure and description

A thermal network makes it possible to distribute the energy produced by one or more heating sources (household waste incineration, boiler, industrial waste that can be recovered) to different users (a district, a town, an industrial zone, etc.) by means of a heat-transfer fluid. Thermal networks are very diversified in size, power and efficiency. They allow the centralization of energy production in order to optimize the overall efficiency of heating installations.

Depending on the heat transfer fluid and the layout of the energy substations, several types or structures of networks can be distinguished: branched networks, meshed networks and mixed networks. In the case of a branched network, the path that connects the power plant to the user is unique. This means that in the case of an intervention between these two points, the user will no longer have access to the energy distributed by the network. In addition, there is usually only one heat source that supplies the network. In the meshed network, there are several possibilities of supplying energy to substations, which can be an advantage as this type of network can have several energy sources. There are also other types of networks that are rather mixed because they have a "master loop" on which small branched networks are connected. Figure 1 shows two variants of branched and meshed network structures for the BlueFACTORY (BF) district in Fribourg.

The network for BlueFACTORY is part of the so-called intelligent 4th generation DH networks. Such type of network must be able to supply heating and cooling energy to all customers while minimizing thermal losses (particularly in heating), to recycle heat from low-temperature sources as well as hot water discharges, to integrate local renewable heat sources (geothermal, solar thermal, ambient air)into the network and, finally, to control and adapt production to the demand.



Fig. 1. BF potential network structures: a) Branched structure, b) Meshed structure

2.2. Distribution network system and terminology

The distribution system includes all parts of the network that are directly between the power plant and the substations. It includes a main or primary circuit comprising Divergents for the distribution of the flow to the Branches and/or Convergents in the case of networks having loops and a secondary circuit comprising Tees to connected substations to the branches.

Figure 2 shows different examples of the structure of branched and meshed networks with different primary and secondary circuits (only the supply circuit of the branches and the substations is shown, although the network also has a return circuit).



(a) 1 branch, 4 substations, 4 segments



(b) 1 branch, 4 substations, 4 segments



(c) 3 branches, 4 substations, 5 segments



(d) 3 branches, 4 substations, 5 segments

Fig. 2. Different structure diagrams: a) In-line branched structure, b) Spike branched structure, c) In-line mesh structure, d) Spike mesh structure

A branch includes all parts of the system that are directly between a source and a Divergent, between two Divergents and between a Divergent and an end valve. A branch can start from the source of the network or from the output of a Divergent. It may end at a Divergent, a Convergent and/or a valve (in the case of branched networks). A branch can be split into different Segments (part of a branch that supplies a substation).

The substations allow heat transfers between the DH network and the consumers. They often consist of heat exchangers or heat pumps for lower temperature networks. In fact, the distribution network provides the amount of energy necessary for the comfort and use of each customer, but for this there is a clear drop in temperature between the inlet and outlet of the substation.

It should be mentioned that pumping systems for the circulation of the flow are not shown. Pumping systems are essential in most of thermal networks. Several cases can be differentiated: a centralized pumping system that has only one pump to deliver the fluid to the source, the network and the substations or a separate pumping system specific to the primary circuit and one specific to the secondary circuit. In the latter case, the aim is to spread the pumping over the entire network, and this

implies greater overall efficiency and greater possibilities if the network is to be expanded. It is essential to choose whether the flow rate should be constant, which implies that the pumping energy is constant and the load is constantly maximum (choice made when the pumping cost is negligible) or whether it should be variable, in which case the thermal power and consumption of the network can be more easily adapted (smart network).

2.3. New approach for representing the distribution network

The simulation of the energy performance of networks makes it possible to determine the distribution of temperature and pressure throughout the network and more particularly to estimate the various network losses (thermal losses, head losses and exergy losses). This consists mainly of representing, modeling and calculating the flow that circulates in each of the branches, segments and substations of the network. Different approaches for the representation and modeling of branched structure networks are proposed, including those also based on graph theory given by Voloshin [5]. The latter uses verticles to represent branches and/or consumers and edges for connections between network points. These methods do not allow the calculation of mesh networks which are in principle more complex to model. A new approach proposed in this context consists in representing any type of network, of any structure, back to a branched network grouping together different substations, segments and/or branches into a network subsystem forming what is called a Cluster. Figure 3 shows a representation graph of a network of clusters. In such a graph, only sources, Divergents and clusters are represented.





Fig. 3. A cluster network graph with 1 Source, 2 Divergents and 7 Clusters

Fig. 4. A branch-type cluster with 2 Divergents, 7 Tees and 7 Substations

Each cluster may itself contain Divergent and/or Convergent, flow distribution or substation supply tees or any other energy source or sink equipment. It can thus be defined by its power (capacity to connect all substations), its efficiency, its level of temperature and pressure and the characteristic magnitudes of the branches and segments that supply its substations. The cluster can also contain branched or meshed circuit elements (refer to the examples of Fig. 2). In the latter case, the presence of a Convergent in the loop is represented by a point on the cluster circle. Figure 4 presents an example of a branch-type cluster including Divergents, tees and substations.

This simplified approach to clustering subsystems makes it possible to manage and model simple or small networks, as well as larger, multi-source energy networks with many substations at different exergy levels. Furthermore, there is no information loss on the details of the substations, their position in the network and the energy carried for their supply.

2.4. Similarities with quantum mechanics

2.4.1. Notion of quantum network based atomic elements

Based on this new formulation, the Cluster is considered as the elementary constituent of the network called atom. It is formed by a group of particles comprising a nucleus (primary network components) and electrons represented by the numerous substations distributed in the secondary network. The nucleus consists of two types of nucleon particles (primary and secondary network): The protons (p) represented by the substation supply tees, which are located at the different starting points of the

secondary network and the Neutrons (n) represented by the Divergent of flow distribution to the branches and segments of the network, located at the primary of the network.

Around these particles are connected electrons (substations). All clusters are made up of these three types of fundamental particles. The number of protons, also known as the atomic number (Z), is also equal to the number of electrons. The mass number (A) represents the sum of the nucleons (n+p). Besides A and Z, the symbol X is used to denote the atomic element $\binom{A}{Z}X$. Figure 5a shows the elementary network cluster according to the above elements.



(a) Detailed elementary network cluster (b) Atomic network cluster with Protons (P) Fig. 5. Representation of elementary network clusters

The nucleus of the atom consists of p=7 protons and n=2 neutrons, giving an atomic mass A=9 (which represents the number of protons and neutrons). The atomic number Z=7 (the number of protons and by extension the number of electrons) and the mass number A are used to designate the cluster or atomic element X. In the case of an atom consisting of a single proton, i.e. Z=1 and A=1 (i.e. a single substation), we designate the element ${}_{1}^{1}X$ by a proton ${}_{1}^{1}P$. Figure 5b shows another form of representing the network by replacing the substations with atomic elements P (in this case, the secondary circuits are not represented). The cohesion of a network is directly related to the type of connections (links) between its particles (neutrons, protons and electron). We can consider two main families of links: the primary links (segments with higher flow rate), which involve neutrons and protons and the secondary links internal to the element (with lower flow rate) which take place mainly between protons and electrons) and links between those particles is called a *quantum network*.

2.4.2. Quantum network isotopes

An element is the set of atoms that have the same atomic number Z. The atoms of an element (even Z) that differ in their mass number A are the isotopes of that element. This concept of network isotope could be used to study network variants in the context of optimization. Figure 6 shows the isotope representations of the different networks of branched and meshed structures presented in Fig. 2 (single-source networks and 4 substations).





(a) branched structure of figures 2a and 2b

(b) meshed structure of figures 2c and 2d

Fig. 6. Isotope diagrams: a) Isotope 4 of element X, 4 protons and 0 neutrons, b) Isotope 5 of element X, 4 protons and 1 neutron

The isotope ${}_{4}^{4}X$ designates the variant of branched structure (line or pin) with 4 substations, therefore having p = 4 protons (4 electrons) and n = 0 neutrons. The isotope ${}_{4}^{5}X$ designates the meshed variant having p = 4 protons, n = 1 neutron and having a point on the circle of the element to specify the presence of a Convergent. This multiple form of representing any network, based either on isotopes

of atomic elements X (clusters), or on links of elements made up of protons P (substations) or both will allow to provide models and perform calculations for complex networks (multi-stage or multi-source, meshed or branched) by taking inspiration from quantum mechanics.

2.5. Quantum network models

The energy levels (or states) that the particles (atoms, neutrons, protons and / or electrons) can take in the network and the different links (branches, segments) which connect them with the main source are described by 3 parameters or quantum numbers. These 3 quantum numbers are called primary quantum number, secondary quantum number and quantum link number.

The primary quantum number (n): this number is represented by the number of neutrons in the network (Divergents). It takes value n = 1, 2, 3, 4, ..., n and will characterize the number of branches on the network. For a given network, there can only be 2n+1 branches. For example, the network presented in Fig. 5, there are mainly 2 neutrons (n = 2) which correspond to 5 branches.

The secondary quantum number (l): this number defines the particular branches in the primary network. It takes the following values: l = -n, ..., -1, 0, +1, ..., + n. With a network of n neutrons, we can write for:

n=0	l = 0	1 branch
n=1	l = -1, 0, +1	3 branches
n=2	l = -2, -1, 0, +1, +2	5 branches
n=3	l = -3, -2, -1, 0, +1, +1	7 branches

The quantum link number (k): this number defines the segment in the branch. It represents the sublayer which is designated by a lowercase letter: s, p, d, f, ... There can be several segments in a branch. The total number of segments in a network is given by the sum of the numbers of neutrons and protons (k = n + p), or by the number of mass A defined above. A particular segment defined by l and k can indeed only lead to a proton or a neutron. The quantum state of a segment is noted $k \cdot l$. Example -3s means that the segment s of the branch -3. Figure 7 shows some examples of networks using the quantum numbers above to specify the location of atoms in different segments. A network pyramid also features next to each variant. It illustrates the link between the Divergent and the branches.

2.5.1. Mass flow balance

The flow conveyed by a Tee (or proton p) \dot{M}_p of the network is equal to the flow delivered to the corresponding substation (or electron e) \dot{M}_e and that the minimum nominal flow conveyed by the source power plant \dot{M}_0 is equal to the sum of the flows of all the substations (electrons or protons).

This rate is constant and is given by the Eq. (1) where Z represents the atomic number (number of substations in the network):

$$\dot{M}_{0} = \sum_{p=1}^{Z} \dot{M}_{p} = \sum_{e=1}^{Z} \dot{M}_{e} .$$
(1)

Likewise, the mass flow rate \dot{M}_L conveyed in any segment of the network (primary link) is equal to the sum of the mass flow rates of the substations (electrons/protons) located downstream.

In this way the mass flow balance could be performed easily on all branches of the network if the position of each segment is known.

Considering the average flow rate of the network as the nominal flow rate of the source \dot{M}_0 in relation to the number Z, (\dot{M}_0/Z), the flow rate of each substation can be expressed as a function of the total flow rate of the source using a flow coefficient μ_e characteristic of the electron, Eq. (2):

$$\dot{M}_e = \mu_e \cdot (\dot{M}_0 / Z) \,. \tag{2}$$

We define the electron load in J/kg as the power of the substation in relation to the total source flow $q_e = \dot{Q}_e / \dot{M}_0$. It represents the proportion of energy from the source exchanged with the substation (electron). Thus, a substation that withdraws energy from the network is an energy sink substation and its load is counted negatively on the total energy in the network. A substation that supplies energy to the network is an energy source substation and its load will be counted positively.

2.5.2. Temperature distribution and heat loss

The overall heat transfer coefficient of a pipe is defined by the Eq. (3),

$$U = \frac{1}{\frac{1}{h_{fluid}} + \frac{r_1 \cdot \ln(r_3 / r_2)}{\lambda_{insulation}} + \frac{r_1 \cdot \ln(r_2 / r_1)}{\lambda_{steel}}}, \quad (3)$$

where h_{fluid} is the heat transfer coefficient determinate by the Gnielinski correlation [14], λ is the thermal conductivity of the pipe and the insulation, r_1 is the inside radius of the pipe, r_2 is the radius at the intersection of metal and insulation and r_3 is the outside radius of the insulation.

The temperature distribution is computed by using a dimensionless factor τ as shown in Eq. (4),

$$\tau = \frac{U \cdot L \cdot d \cdot \pi}{\dot{M} \cdot c_p} = \frac{T_i - T_o}{\frac{T_i + T_o}{2} - T_a},$$
(4)

where U is the overall heat transfer coefficient of the pipe, L is the length of the pipe, d is the inside

diameter of the pipe, m is the mass flow rate and c_p is the thermal mass capacity of the fluid. The simplification is done with the Eq. (5),

$$\frac{T_i - T_a}{T_o - T_a} = \frac{1 + \tau / 2}{1 - \tau / 2},$$
(5)

where T_i and T_o are the inlet and outlet temperatures of the pipe and T_a is the ambient temperature of the environment. Then, the heat loss Q of the pipe is calculated by the Eq. (6),

$$\dot{Q}_{loss} = \dot{M} \cdot c_p \cdot (T_i - T_o) \,. \tag{6}$$

2.5.3. Pressure drops

Linear pressure losses ΔP_l is calculated as follows with the Eq. (7),

$$\Delta P_l = \rho \cdot f \cdot (L/d) \cdot v^2 / 2, \qquad (7)$$

where ρ is the density of the fluid, f is the friction factor determined with the Goudar–Sonnad equation [15] as function of the pipe's roughness and Reynold's number, L is the length of the pipe, d is the diameter of the pipe and v is the fluid velocity inside the pipe.

3. Application of the method

3.1. Variants and procedure

In this chapter the method will be applied to four different networks in Fig. 7 whose particularity is that the substations are located in the same place and that the energy consumed by each one is identical between each version. The maximum fluid velocity in each pipe is 1-2 m/s.

In order to determine the flow, pressure, and temperature conditions in each segment the following procedure was followed. The annual energy for each substation is first distributed according to the outdoor temperature as well as the type of building on an hourly basis. This makes it possible to obtain the necessary mass flow rate for each substation and therefore the flow rate of each point of

the network is known by the Eq. (1, 2). The heat transfer coefficient and the τ factor can thus be calculated for each segment by the Eq. (3, 4).



Fig. 7. Four versions of 8 substations: a) 8 protons, 3 neutrons, b) 8 protons, 3 neutrons, c) 8 protons, 2 neutrons, d) 6 protons, 1 atom of 2 protons (loop), 2 neutrons

The temperature distribution in the network is computed for each pipe inlet and outlet by the Eq. (5) beginning with the segment at the power plant outlet where the temperature is set. Finally, the heat losses and the linear pressure losses are determined using the Eq. (6, 7). Singular pressure losses (exchangers, valves, elbows, tees) are estimated with conventional models.



Fig. 8. Characteristics of the (a) variant: a) DHN power, b) DHN pressure drops

Since the conditions are known for the entire network, a monitoring of the latter is carried out as illustrated below. Figure 8a shows the classified power curves over the year. The nominal power of the DHN is 2970 MW and heat losses vary from 2.4% to 22.2% of the supply power during the year.

¹ DH network (*d*) simplifies two substations (protons) into one atom

The pressure loss curves are shown in Fig. 8b for a maximum pressure loss of 2090 kPa. The summer period (\sim 3000h) when only domestic hot water production is required is clearly visible on the right-hand side of the graphs.

The hourly evolution of conditions in the first segment of variants (a), (b), and (d) is shown in Fig. 9. As expected, the parameters reach their minima during the summer (between 3500 and 6000 hours) when heat consumption is lowest.



Fig. 9. Monitoring of the mass flow rate, pressure drop and heat loss of the first segment (a, b, d)

These results are obtained by considering steady-state conditions for each time step. The axial thermal conduction of the fluid and the pipe wall can be ignored because they are very small compared with the radial heat conduction. The heat storage of the fluid, adjacent pipe and fittings is negligible.

3.2. Results and comparison

Simulation results for the four variants are shown in Table 1 below. The comparison of the four variants in Fig. 7 will be made with the eight parameters shown in Table 1.

Variant	а	b	С	d
Network supply length, m	2190	1750	1830	1610^{*}
Nominal mass flow rate, kg/s	49.9	49.9	49.9	49.9
\dot{M}_0/Z	6.2	6.2	6.2	7.1
Supply energy, MWh/y	7550	7452	7533	7435*
Heat demand, MWh/y	7000	7000	7000	7000
Heat loss, MWh/y	550	452	533	435*
Heat loss ratio, %	7.3	6.1	7.1	5.8^{*}
Nominal pressure drops, bar	20.8	20.6	11.7	14.2^{*}
Nominal linear pressure drops, bar	13.6	12.6	7.3	9.0^{*}

Table 1. Comparison of four DH networks

**Reduced network*

Although variant (a) serves the large heat consumers before the small ones, as the path is by far the longest, the heat losses as well as the pressure drops are the highest. Variant (b) is the most optimal from the point of view of heat loss with a ratio of 6.1% of the total annual energy. Variant (d) achieves an even lower score (5.8%) however some 200 m of pipe has been simplified by combining the P2 and P5 consumers into one atom.

Variant (c), which uses only 2 Divergent (neutrons), carries high flow rates throughout the network compared to the variants where a Divergent is present right at the outlet of the plant. This configuration requires larger pipe diameters which results in lower pressure drops but higher heat losses compared to a system of similar size (e.g. variant (b)). This explains why variant (c) is the best from the standpoint of pressure loss. The operating conditions of variant (d) seem optimal, but as mentioned above, this network contains an atom whose loop has been processed prior to this calculation. The conditions associated with this simplification are therefore at a lower resolution level than that considered in this chapter.

4. Case study: BlueFACTORY

The DH network of the BlueFACTORY (BF) district in Fribourg illustrated by Fig. 10 is able to provide a power of 3.2 MW at 80°C for an annual energy supply of 7700 MWh to thirteen consumers. The network reaches a length of 540 m through thirteen branches and six Divergent. As the energy supplied is substantial in relation to the length of the network, it is a very high energy density network. The energy density value reaches 14.3 MWh/m.



Fig. 10. BlueFACTORY's DH network in Fribourg, Switzerland

These characteristics are clearly identifiable in Fig. 11a. Indeed, as the energy density is high, the ratio of heat losses to power supplied is much lower 0.6-6.4% than the networks illustrated in Chapter 4 (density of 3.4-4.3 MWh/m). Another characteristic of short DH network supplying many consumers is that the singular pressure drops are preponderant over the linear losses as shown in Fig. 11b.



Fig. 11. Characteristics of BlueFACTORY's DHN: a) DHN power, b) DHN pressure drops

As in Chapter 4, Fig. 12 shows the hourly variation of mass flow rate, pressure drop and heat loss conditions in the first segment of the DH network. The heat loss on this segment is quite low because the length is only 3 m.

This network is intended to welcome other users in the future. However, the capacity of the power plant should not change drastically since the objective is to reduce the temperature level of the network in order to reach medium temperature levels (30-40°C). The buildings on the site will all be renovated, the lowering of the temperature level will not cause any problem and the production of domestic hot water could be done in a decentralized system or by heat pump in substation. As a result, it is expected that the system's exergy efficiency will be significantly improved in the future.



Fig. 12. Monitoring of the mass flow rate, pressure drop and heat loss of BF networks first segment

5. Conclusion

A new approach based on a so called "quantum network" is proposed of representing any type of network, of any structure. It consists of grouping together different substations, segments and/or branches into a network subsystem forming what is called a Cluster. Based on this new formulation, the Cluster is considered as an elementary constituent of a network called atom. It is formed by a group of particles comprising a nucleus (primary network components) and electrons represented by the numerous substations distributed in the secondary network. This method allows to perform a general network modelling with greater accuracy of simulation results, resulting in significant cost savings. A difference of a few tenths of a percent in heat loss can have a large impact on the operating costs of a network over the years. The method presented in this article meets these criteria while at the same time making the calculation process more efficient. It has been applied to four different networks in order to demonstrate the robustness of the quantum network models and compare the performances of such networks. A typical application for a set of high-performance buildings in the BlueFACTORY district (Fribourg, Switzerland) has been also considered to simulate different structures (branched, mesh or mixed network systems) of advanced thermal networks. The performances, in term of pressure drop and thermal losses on pipes, are determined in hourly basis.

Nomenclature

Latin symbols

- A atomic mass
- Z atomic number
- U overall heat transfer coefficient, W/(m² K)
- *h* heat transfer coefficient, $W/(m^2 K)$
- r radius, m
- L length, m
- d diameter, m

M mass flow rate, kg s⁻¹

- q load, J/kg
- c specific heat, J/(kg K)
- T temperature, K
- \dot{Q} heat power, W
- ΔP Pressure drops, Pa

- f friction factor
- v velocity, m s⁻¹

Greek symbols

- μ characteristic of the electron
- λ thermal conductivity, W/(m K)
- τ dimensionless pipe factor
- ρ fluid density, kg/m³

Subscripts and superscripts

- 0 nominal
- p proton
- e electron
- a ambient
- i inlet
- o outlet
- l linear

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